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| Please check the examination details below before entering your candidate information |  |                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                     |
| Candidate surname                                                                     |  | Other names                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                     |
| <b>Pearson Edexcel</b><br><b>Level 3 GCE</b>                                          |  | Centre Number                                                                                                                                                                                                                                                                                                                                                                                                        | Candidate Number                                                                                                                                                                                                                                                                                                                    |
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| <b>Wednesday 5 June 2019</b>                                                          |  |                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                     |
| Morning (Time: 2 hours)                                                               |  | Paper Reference <b>9MA0/01</b>                                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                     |
| <b>Mathematics</b><br><b>Advanced</b><br><b>Paper 1: Pure Mathematics 1</b>           |  |                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                     |
| <b>You must have:</b><br>Mathematical Formulae and Statistical Tables, calculator     |  |                                                                                                                                                                                                                                                                                                                                                                                                                      | Total Marks<br><div style="border: 1px solid black; width: 50px; height: 50px; margin: 0 auto;"></div>                                                                                                                                                                                                                              |

**Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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Answer ALL questions. Write your answers in the spaces provided.

1.

$$f(x) = 3x^3 + 2ax^2 - 4x + 5a$$

Given that  $(x + 3)$  is a factor of  $f(x)$ , find the value of the constant  $a$ .

(3)

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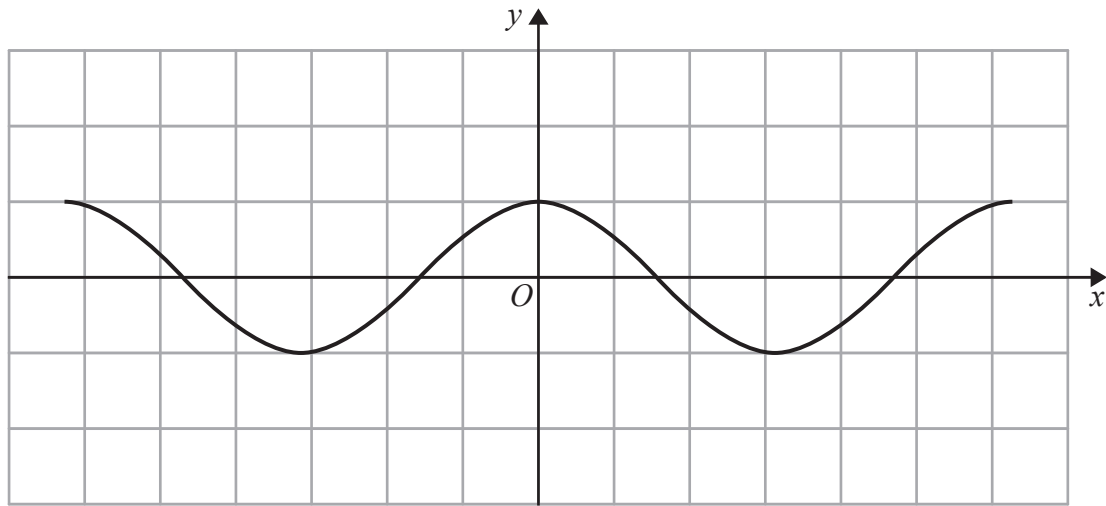
Question 1 continued

Lined area for writing the answer to Question 1.

(Total for Question 1 is 3 marks)



2.



### Figure 1

Figure 1 shows a plot of part of the curve with equation  $y = \cos x$  where  $x$  is measured in radians. Diagram 1, on the opposite page, is a copy of Figure 1.

- (a) Use Diagram 1 to show why the equation

$$\cos x - 2x - \frac{1}{2} = 0$$

has only one real root, giving a reason for your answer.

(2)

Given that the root of the equation is  $\alpha$ , and that  $\alpha$  is small,

- (b) use the small angle approximation for  $\cos x$  to estimate the value of  $\alpha$  to 3 decimal places. (3)

(3)



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Question 2 continued

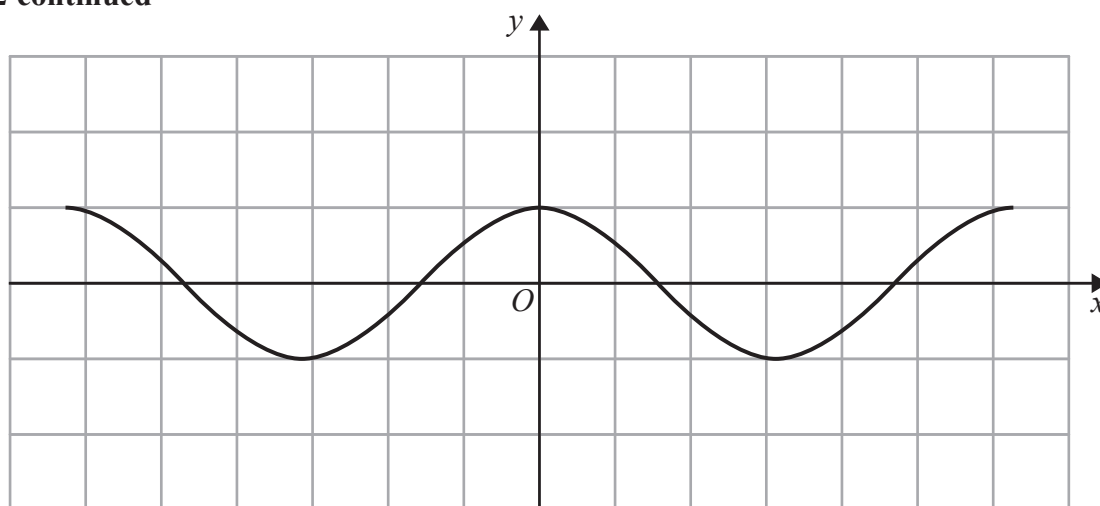


Diagram 1

(Total for Question 2 is 5 marks)



3.

$$y = \frac{5x^2 + 10x}{(x+1)^2} \quad x \neq -1$$

(a) Show that  $\frac{dy}{dx} = \frac{A}{(x+1)^n}$  where  $A$  and  $n$  are constants to be found. (4)

(b) Hence deduce the range of values for  $x$  for which  $\frac{dy}{dx} < 0$  (1)



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4. (a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

(4)

The expansion can be used to find an approximation to  $\sqrt{2}$

Possible values of  $x$  that could be substituted into this expansion are:

- $x = -14$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

- (b) Without evaluating your expansion,

- (i) state, giving a reason, which of the three values of  $x$  should not be used

(1)

- (ii) state, giving a reason, which of the three values of  $x$  would lead to the most accurate approximation to  $\sqrt{2}$

(1)





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5.

$$f(x) = 2x^2 + 4x + 9 \quad x \in \mathbb{R}$$

(a) Write  $f(x)$  in the form  $a(x + b)^2 + c$ , where  $a$ ,  $b$  and  $c$  are integers to be found. (3)

(b) Sketch the curve with equation  $y = f(x)$  showing any points of intersection with the coordinate axes and the coordinates of any turning point. (3)

(c) (i) Describe fully the transformation that maps the curve with equation  $y = f(x)$  onto the curve with equation  $y = g(x)$  where

$$g(x) = 2(x - 2)^2 + 4x - 3 \quad x \in \mathbb{R}$$

(ii) Find the range of the function

$$h(x) = \frac{21}{2x^2 + 4x + 9} \quad x \in \mathbb{R} \quad (4)$$



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Question 5 continued

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**(Total for Question 5 is 10 marks)**



6. (a) Solve, for  $-180^\circ \leq \theta \leq 180^\circ$ , the equation

$$5 \sin 2\theta = 9 \tan \theta$$

giving your answers, where necessary, to one decimal place.

*[Solutions based entirely on graphical or numerical methods are not acceptable.]*

(6)

- (b) Deduce the smallest positive solution to the equation

$$5 \sin (2x - 50^\circ) = 9 \tan (x - 25^\circ)$$

(2)



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Question 6 continued

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Question 6 continued

Lined area for writing answers.

(Total for Question 6 is 8 marks)



7. In a simple model, the value, £ $V$ , of a car depends on its age,  $t$ , in years.

The following information is available for car  $A$

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car  $A$ , a possible equation linking  $V$  with  $t$ . (4)

The value of car  $A$  is monitored over a 10-year period.

Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information. (2)

The following information is available for car  $B$

- it has the same value, when new, as car  $A$
- its value depreciates more slowly than that of car  $A$

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car  $B$ . (1)



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Question 7 continued

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**(Total for Question 7 is 7 marks)**



8.

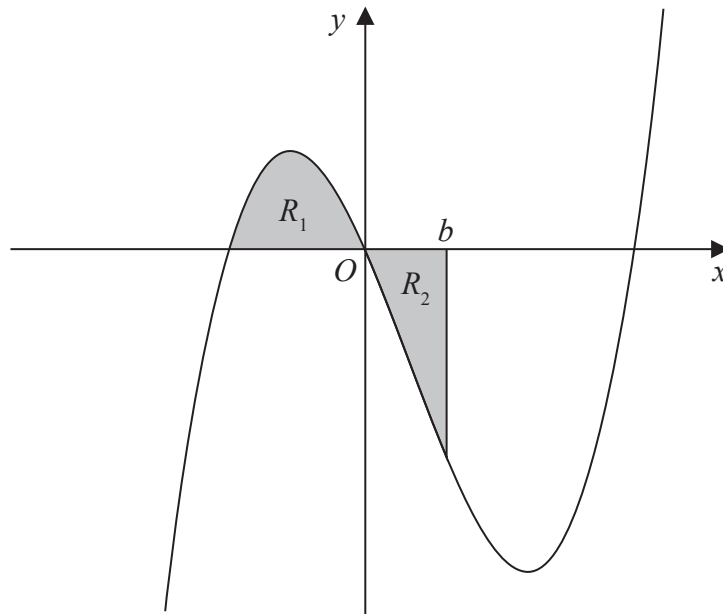


Figure 2

Figure 2 shows a sketch of part of the curve with equation  $y = x(x + 2)(x - 4)$ .

The region  $R_1$  shown shaded in Figure 2 is bounded by the curve and the negative  $x$ -axis.

- (a) Show that the exact area of  $R_1$  is  $\frac{20}{3}$  (4)

The region  $R_2$  also shown shaded in Figure 2 is bounded by the curve, the positive  $x$ -axis and the line with equation  $x = b$ , where  $b$  is a positive constant and  $0 < b < 4$

Given that the area of  $R_1$  is equal to the area of  $R_2$

- (b) verify that  $b$  satisfies the equation

$$(b + 2)^2 (3b^2 - 20b + 20) = 0 \quad (4)$$

The roots of the equation  $3b^2 - 20b + 20 = 0$  are 1.225 and 5.442 to 3 decimal places. The value of  $b$  is therefore 1.225 to 3 decimal places.

- (c) Explain, with the aid of a diagram, the significance of the root 5.442 (2)



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Question 8 continued

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Question 8 continued

Handwriting practice lines for Question 8.

(Total for Question 8 is 10 marks)



9. Given that  $a > b > 0$  and that  $a$  and  $b$  satisfy the equation

$$\log a - \log b = \log(a - b)$$

- (a) show that

$$a = \frac{b^2}{b - 1} \quad (3)$$

- (b) Write down the full restriction on the value of  $b$ , explaining the reason for this restriction. (2)



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Question 9 continued

Handwriting practice area with horizontal lines.

(Total for Question 9 is 5 marks)



10. (i) Prove that for all  $n \in \mathbb{N}$ ,  $n^2 + 2$  is not divisible by 4

(4)

(ii) "Given  $x \in \mathbb{R}$ , the value of  $|3x - 28|$  is greater than or equal to the value of  $(x - 9)$ ."

State, giving a reason, if the above statement is always true, sometimes true or never true.

(2)



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**(Total for Question 10 is 6 marks)**



11. A competitor is running a 20 kilometre race.

She runs each of the first 4 kilometres at a steady pace of 6 minutes per kilometre.  
After the first 4 kilometres, she begins to slow down.

In order to estimate her finishing time, the time that she will take to complete each subsequent kilometre is modelled to be 5% greater than the time that she took to complete the previous kilometre.

Using the model,

- (a) show that her time to run the first 6 kilometres is estimated to be 36 minutes 55 seconds, (2)

- (b) show that her estimated time, in minutes, to run the  $r$ th kilometre, for  $5 \leq r \leq 20$ , is

$$6 \times 1.05^{r-4} \quad (1)$$

- (c) estimate the total time, in minutes and seconds, that she will take to complete the race. (4)



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Question 11 continued

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**(Total for Question 11 is 7 marks)**



12.  $f(x) = 10e^{-0.25x} \sin x, \quad x \geq 0$

- (a) Show that the  $x$  coordinates of the turning points of the curve with equation  $y = f(x)$  satisfy the equation  $\tan x = 4$

(4)

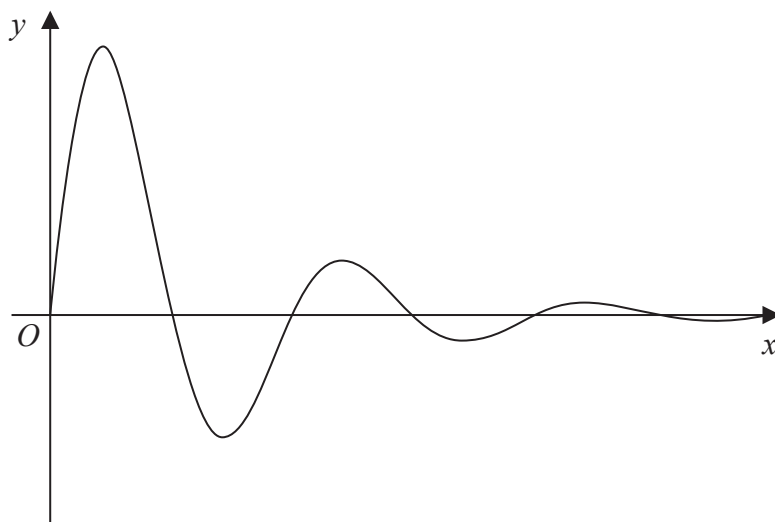


Figure 3

Figure 3 shows a sketch of part of the curve with equation  $y = f(x)$ .

- (b) Sketch the graph of  $H$  against  $t$  where

$$H(t) = |10e^{-0.25t} \sin t| \quad t \geq 0$$

showing the long-term behaviour of this curve.

(2)

The function  $H(t)$  is used to model the height, in metres, of a ball above the ground  $t$  seconds after it has been kicked.

Using this model, find

- (c) the maximum height of the ball above the ground between the first and second bounce.

(3)

- (d) Explain why this model should not be used to predict the time of each bounce.

(1)

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Question 12 continued

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**(Total for Question 12 is 10 marks)**



13. The curve  $C$  with equation

$$y = \frac{p - 3x}{(2x - q)(x + 3)} \quad x \in \mathbb{R}, x \neq -3, x \neq 2$$

where  $p$  and  $q$  are constants, passes through the point  $\left(3, \frac{1}{2}\right)$  and has two vertical asymptotes with equations  $x = 2$  and  $x = -3$

(a) (i) Explain why you can deduce that  $q = 4$

(ii) Show that  $p = 15$

(3)

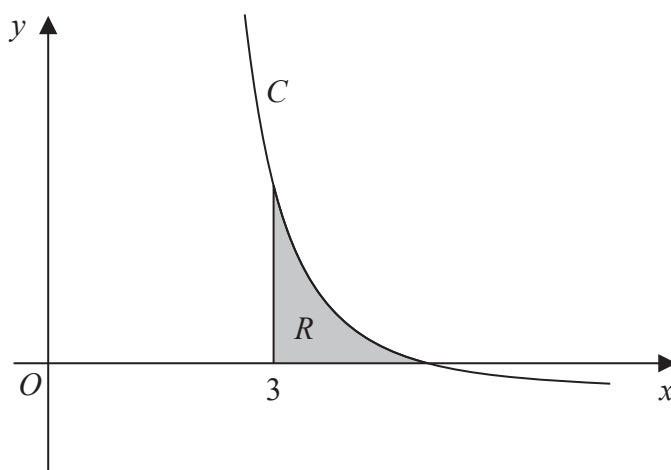


Figure 4

Figure 4 shows a sketch of part of the curve  $C$ . The region  $R$ , shown shaded in Figure 4, is bounded by the curve  $C$ , the  $x$ -axis and the line with equation  $x = 3$

(b) Show that the exact value of the area of  $R$  is  $a \ln 2 + b \ln 3$ , where  $a$  and  $b$  are rational constants to be found.

(8)



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Question 13 continued

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**(Total for Question 13 is 11 marks)**



14. The curve  $C$ , in the standard Cartesian plane, is defined by the equation

$$x = 4 \sin 2y \quad -\frac{\pi}{4} < y < \frac{\pi}{4}$$

The curve  $C$  passes through the origin  $O$

(a) Find the value of  $\frac{dy}{dx}$  at the origin. (2)

(b) (i) Use the small angle approximation for  $\sin 2y$  to find an equation linking  $x$  and  $y$  for points close to the origin.

(ii) Explain the relationship between the answers to (a) and (b)(i). (2)

(c) Show that, for all points  $(x, y)$  lying on  $C$ ,

$$\frac{dy}{dx} = \frac{1}{a\sqrt{b-x^2}}$$

where  $a$  and  $b$  are constants to be found. (3)



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Question 14 continued

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**Question 14 continued**

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**(Total for Question 14 is 7 marks)**

**TOTAL FOR PAPER IS 100 MARKS**

